Optimized arrays for resistivity measurements confined to the perimeter of a survey area

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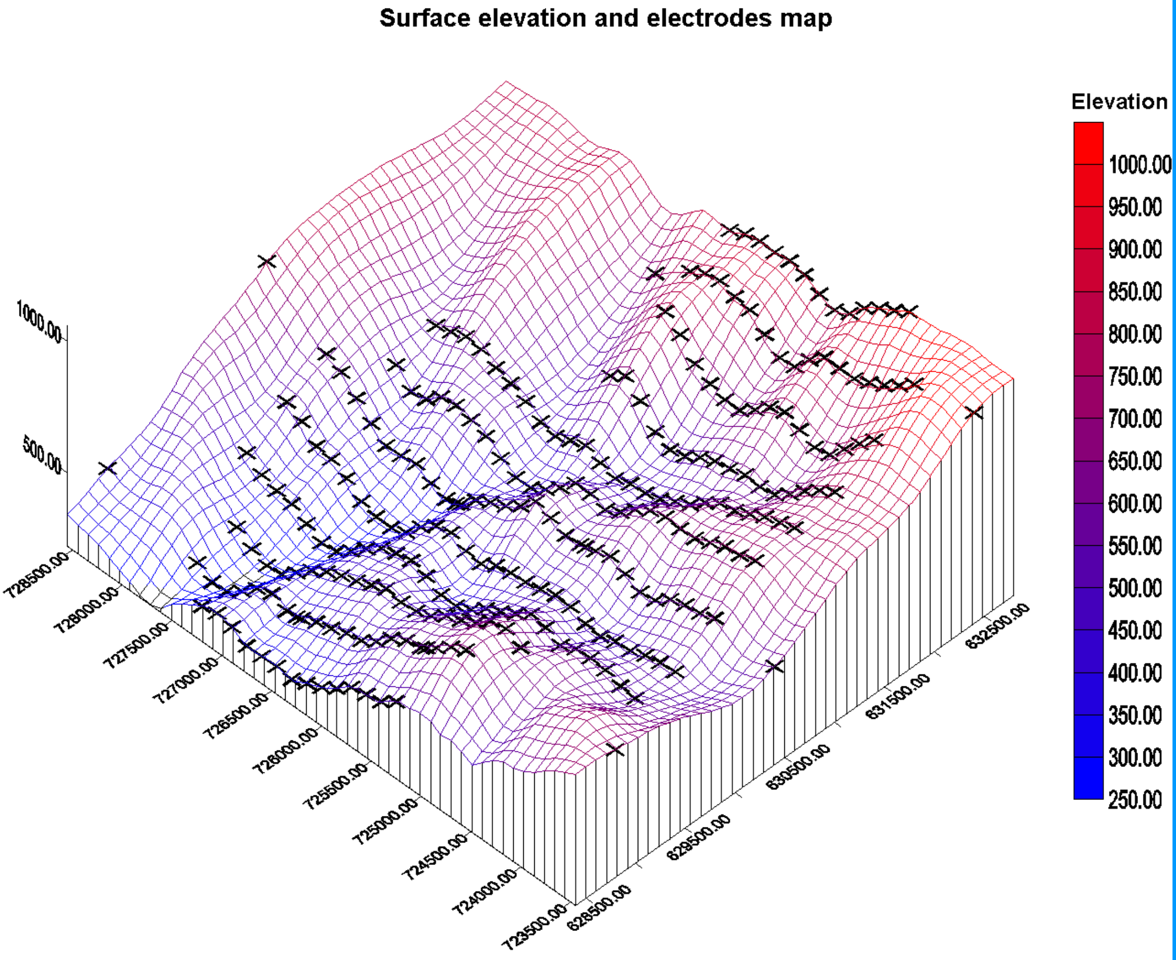
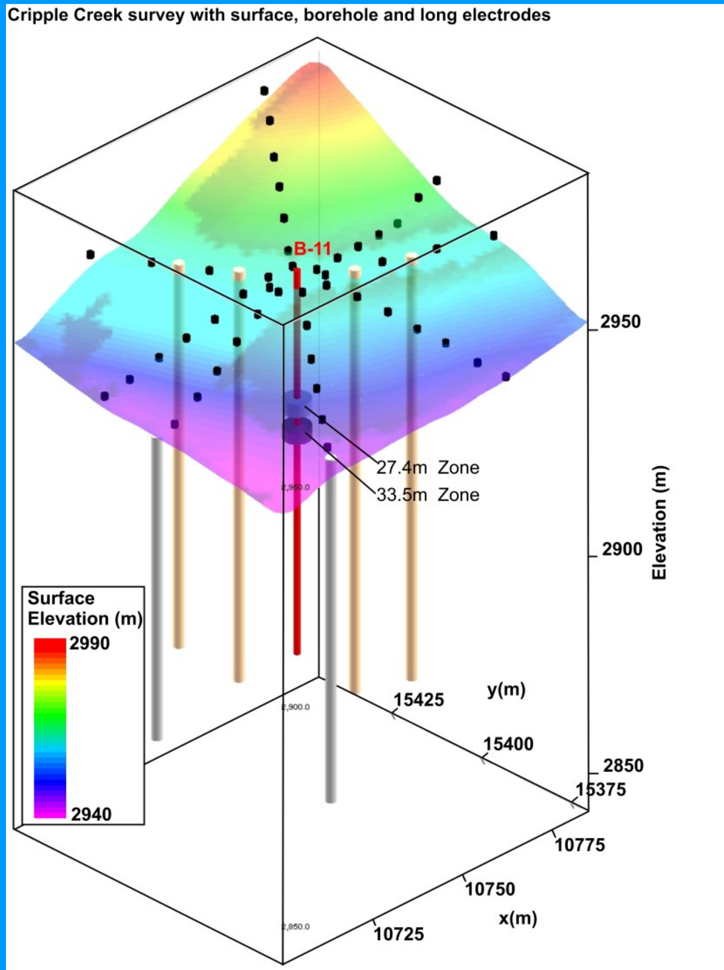
Outline

1. 3-D Surveys along a perimeter
2. Optimized arrays
3. Example with a rectangular perimeter
4. Example with a circular perimeter
5. Conclusions

'Normal' 3-D surveys

Most 3-D surveys attempt to cover the survey area using roughly parallel lines or more complex arrangements. This sometimes includes subsurface electrodes to obtain better resolution for structures under the area of interest.



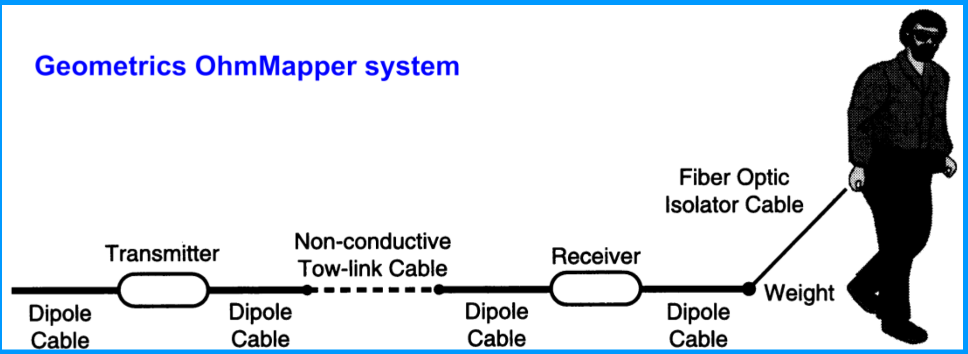


‘Restricted’ 3-D surveys

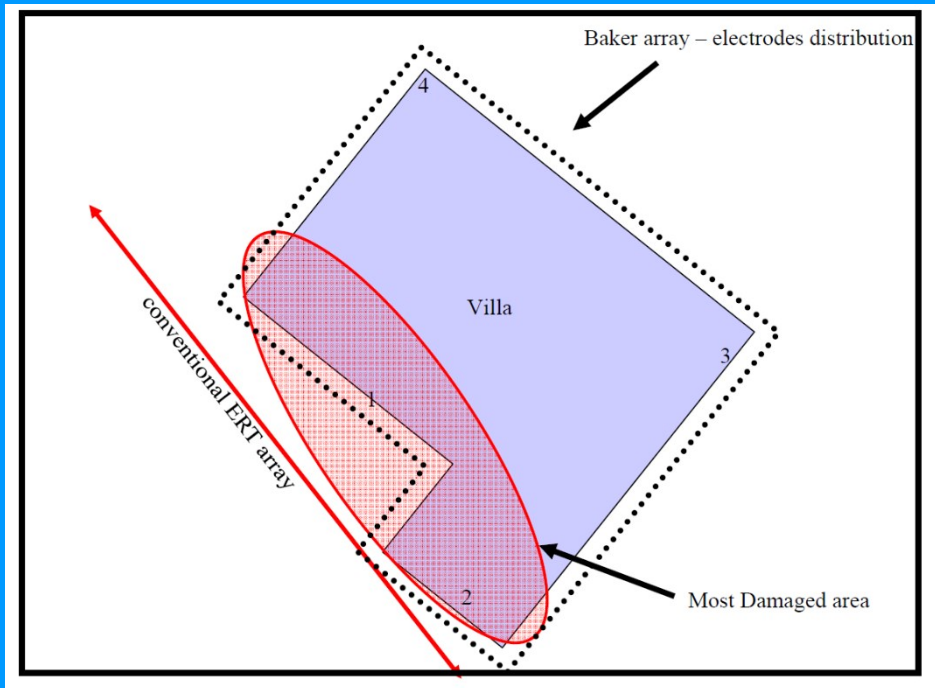
Some built-up areas are covered, and it is not possible to use normal implanted electrodes.

One solution - use of non-contact capacitively coupled resistivity meters. Problems – expensive equipment, not widely available.

Practical solution – plant electrodes around perimeter of survey area.



Benhammam, K., Baker, H. and Nuaimi, M.R. Al., 2011. Delineating Cavities Underneath Constructed Sites – Case Study in Al Ain CityNormal access. First EAGE International Conference on Engineering Geophysics, Al Ain, United Arab Emirates.

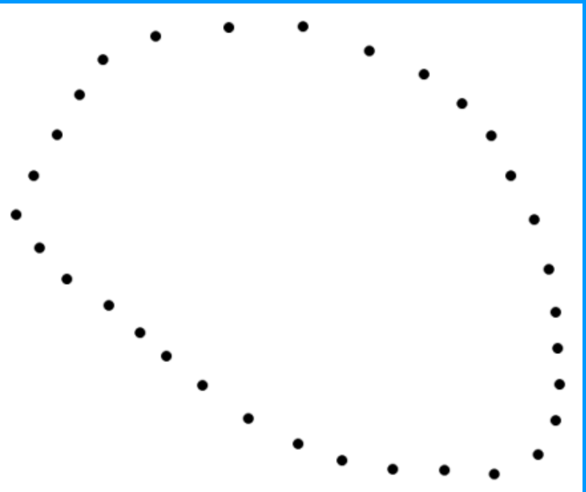


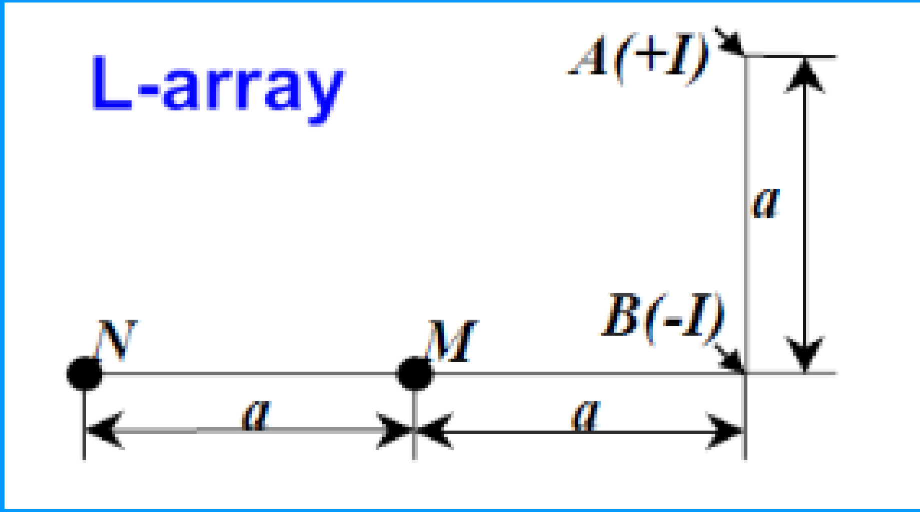
Arrays to use

Problem of resolution – as the position of the electrodes are restricted, resolution is expected to be poorer than a 'normal' 3-D survey.

Heurestic rules – Baker, L, corner arrays. Usually designed for perimeters with sharp corners.

Do these arrays provide the best possible resolution? What about odd-shaped perimeters? A general method that can be used for perimeters of different shapes?





The least-squares method and model resolution

The smoothness-constrained least-squares method :-

$$\left(\mathbf{G}^{T}\mathbf{G}+\lambda\mathbf{C}\right)\Delta\mathbf{q}_{i}=\mathbf{G}^{T}\mathbf{g}-\lambda\mathbf{C}\mathbf{q}_{i-1}$$

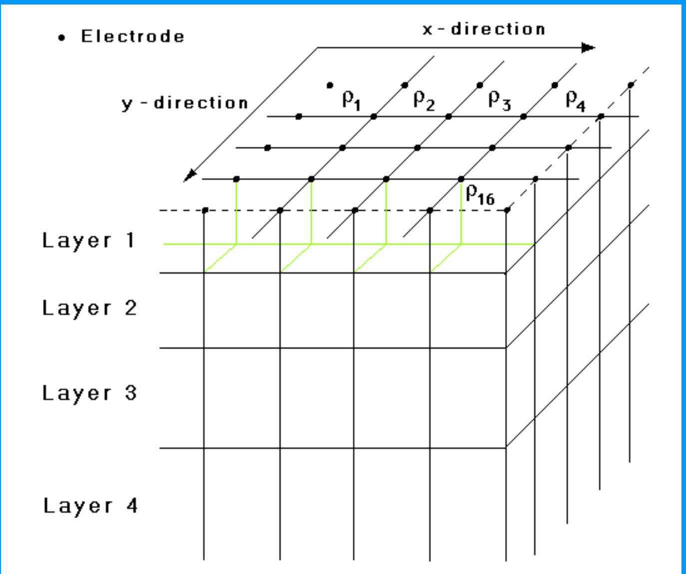
C = roughness filter, **λ** = damping factor, **q** = model resistivity, **g** = data misfit. **J** = Jacobian matrix of partial derivatives. The model resolution matrix **R** is given by

$$\mathbf{R}=\mathbf{B}\ \mathbf{A},\quad \text{where}\ \mathbf{A}=\mathbf{G}^{T}\mathbf{G}\ \text{and}\ \mathbf{B}=\left(\mathbf{A}+\lambda\mathbf{C}\right)^{-1}$$

The main diagonal of **R** give the model cells resolutions.

R can be considered as a 'filter' through which we see the subsurface.

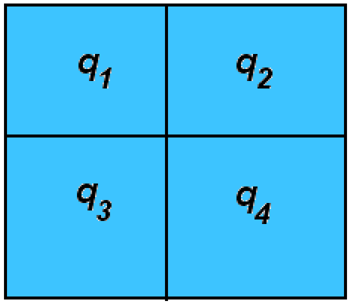
$$\mathbf{q}_{\text{Model}}\approx\mathbf{R}\,\mathbf{q}_{\text{Actual}}$$



$$\begin{pmatrix} q_{M1}\\ q_{M2}\\ q_{M3}\\ q_{M4}\end{pmatrix}=\begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14}\\ R_{21} & R_{22} & R_{23} & R_{24}\\ R_{31} & R_{32} & R_{33} & R_{34}\\ R_{41} & R_{42} & R_{43} & R_{44}\end{pmatrix}\begin{pmatrix} q_{A1}\\ q_{A2}\\ q_{A3}\\ q_{A4}\end{pmatrix}$$

$$\mathbf{q}_{\text{Model}}=\mathbf{R}\quad\mathbf{q}_{\text{Actual}}$$

Model with 4 cells



The ‘Compare R’ array optimization method

Select arrays that give the maximum model resolution.

1). Create a ‘comprehensive’ data set that contains all the possible ‘stable’ arrays. Reject arrays with geometric factors above a selected limit (eg. DD a=1,n=10), and are sensitive to errors in electrode positions.

2). Start with a small ‘base’ data set, such as dipole-dipole array with n=1 and a=1.

3). Calculate change in model resolution when a new array from the comprehensive data set is added to the base data set. Select the arrays that give the largest increase in the model resolution.

Calculting the change in the resolution

The change in the model resolution ΔR when a new array is added is calculated using the following equation.

$$\Delta\mathbf{R} = \frac{\mathbf{z}}{1+\mu}\left(\mathbf{g}^{\mathrm{T}} - \mathbf{y}^{\mathrm{T}}\right), \ \ where\ \mathbf{y} = \mathbf{A}\mathbf{z},\ \mathbf{z} = \mathbf{B}\mathbf{g},\ \mu = \mathbf{g}.\mathbf{z}$$

g is sensitivity vector for the array. The improvement in the resolution is given by a weighted sum of the change in the resolution of the base set divided by the comprehensive set resolution.

$$F_{CR} = \frac{1}{m}\sum_{j=1}^{j=m} c(j).\Delta R_b(j,j) / R_c(j,j)$$

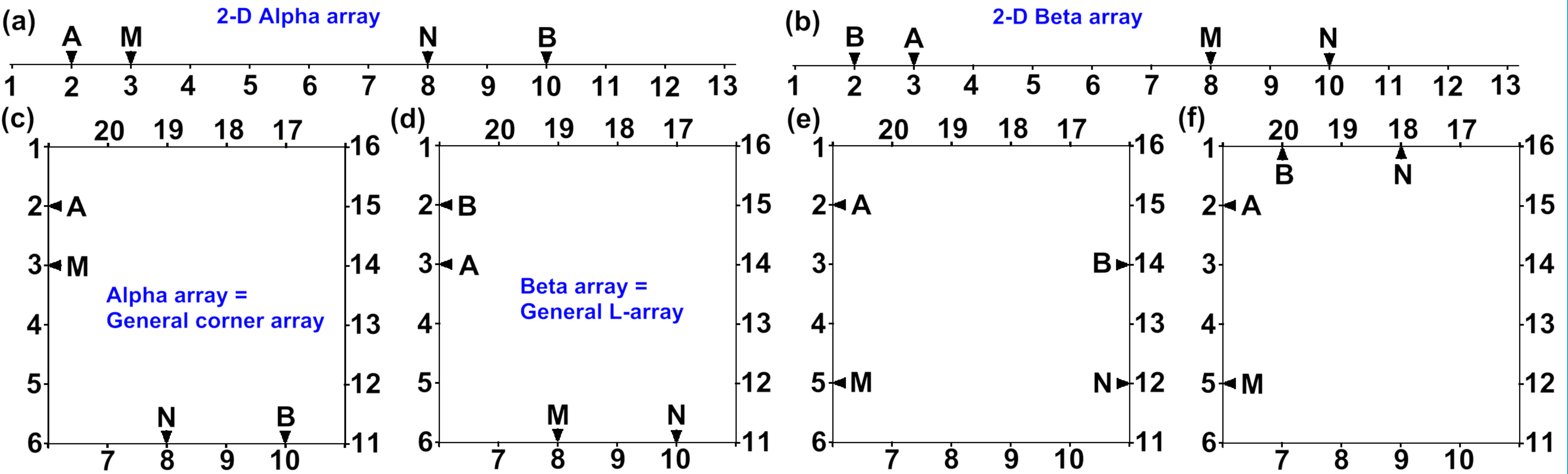
R_c = resolution of comprehensive set, ΔR_b = change in resolution of base set. $c(j)$ is a weighting function to give more weight to the model cells within the survey area. The arrays with the largest F_{CR} values are selected.

The possible arrays in comprehensive data set

Modify the method used for a 2-D line. Start with electrodes on left end of line and move step by step to right end of line, except line is now a closed loop. Find all possible combinations of ‘alpha’ and ‘beta’ arrays that are stable.

Alpha arrays = includes all ‘corner arrays’

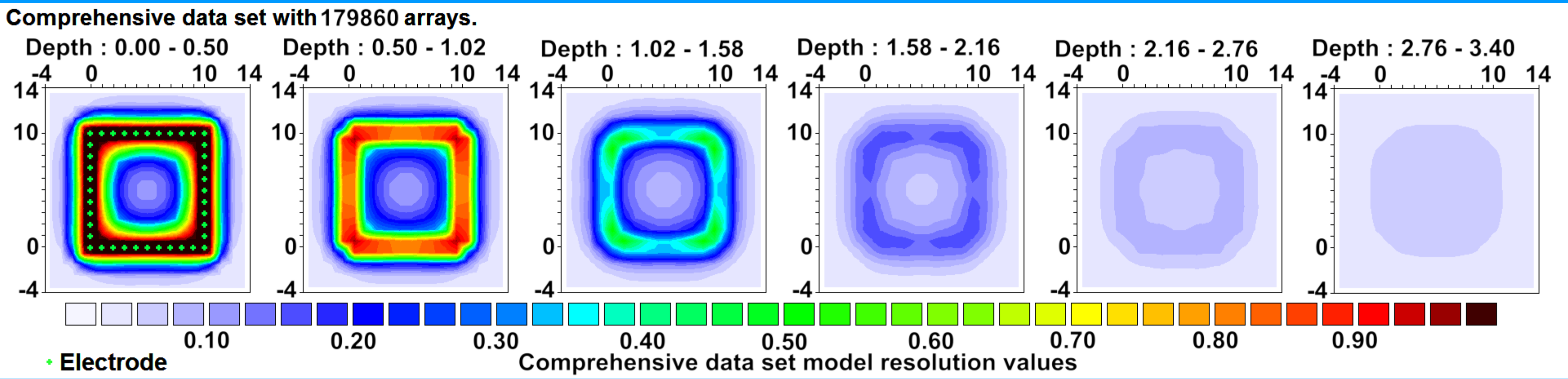
Beta arrays = includes all ‘L-arrays’



Example of square perimeter

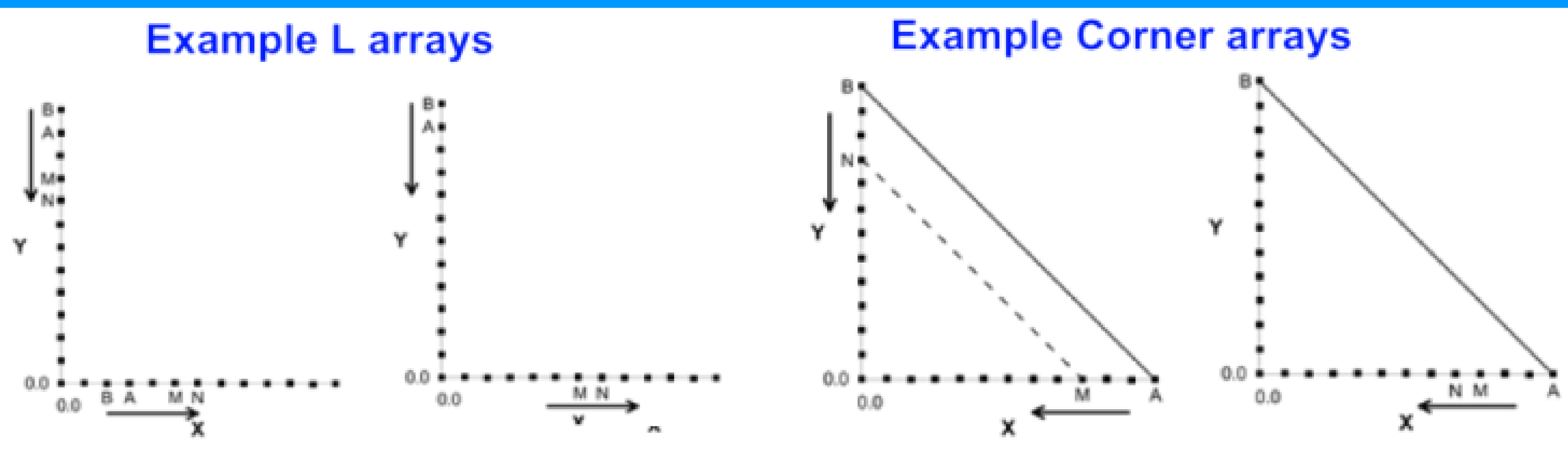
Survey with 40 electrodes in a square perimeter. There are 179860 possible alpha and beta arrays. First we examine the model resolution for the comprehensive data set. The resolution decreases towards center of survey area and with depth. Symmetrical about center.

Since the ‘comprehensive’ data set contains all the possible arrays (with maximum geometric factor of 4147 m), it shows the maximum resolution that can be achieved by the survey setup.



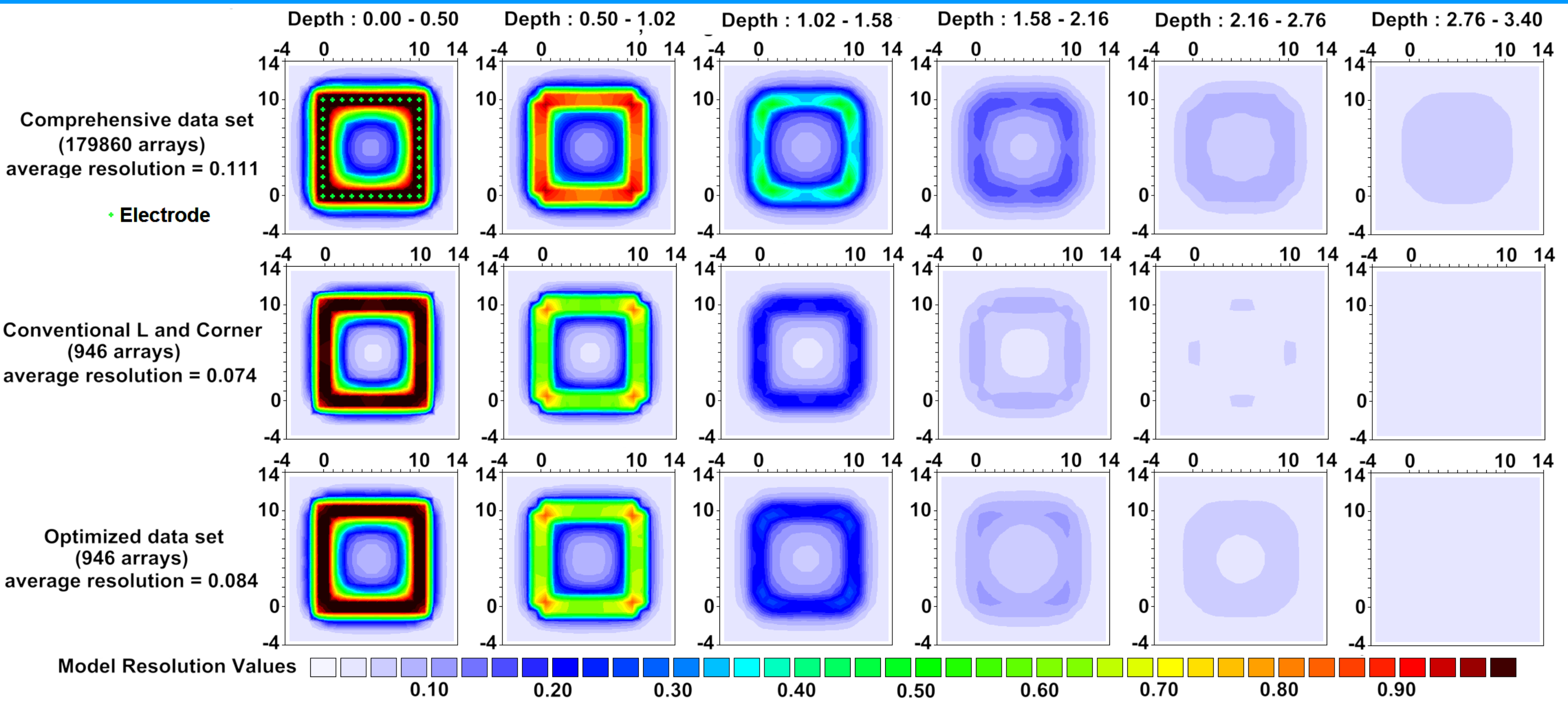
‘Conventional’ test arrays

The ‘conventional’ arrays used for comparison consists of the ‘L’ and ‘Corner’ types. It has 946 data points for a square perimenter with 40 electrodes. The ‘L’ array has the M-N potential dipole separate from the A-B current dipole, it is a ‘Beta’ type array. Th ‘Corner’ array is an ‘Alpha’ type array. The comprehensive data set used for array optimization includes non-symmetrical versions with unequal dipoles as well.



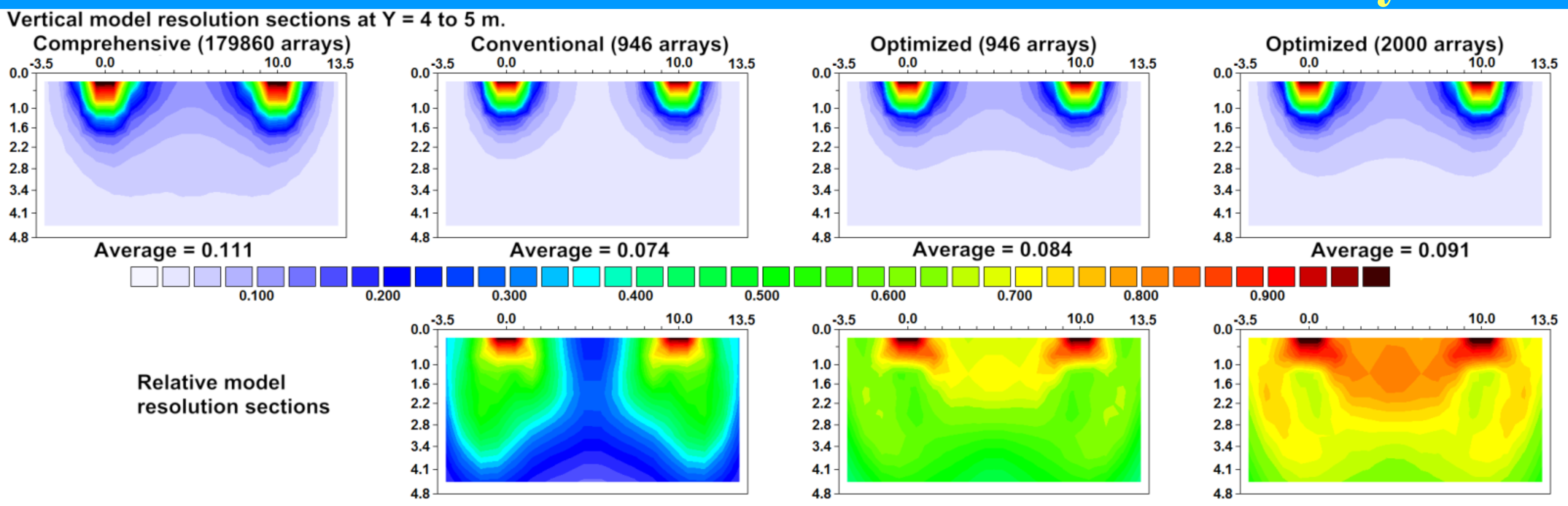
Comparison of conventional and optimized arrays

The comprehensive data set has significant resolution until the 6th layer, while the smaller optimized data set reaches until the 5th layer. The optimized data set has better resolution compared to the conventional arrays towards the center and in the deeper layers.



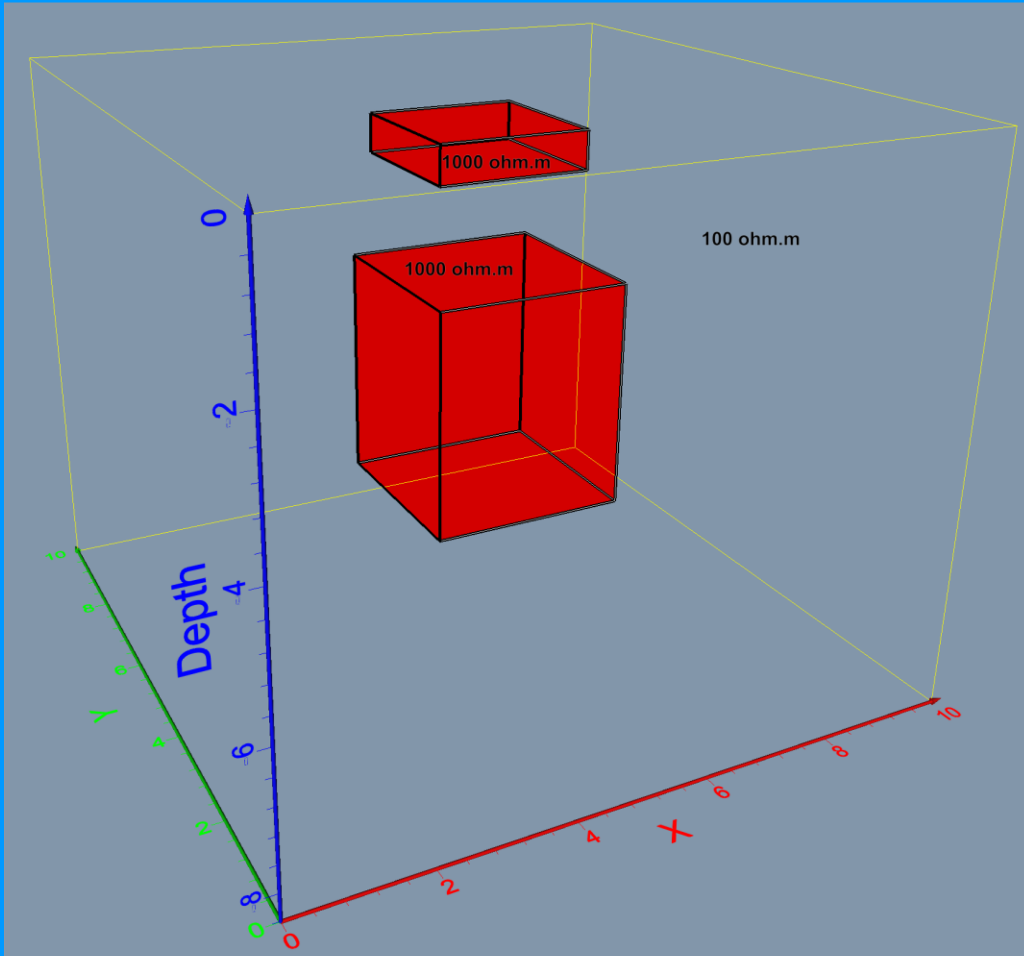
Comparison of conventional and optimized arrays

The model resolution sections in a vertical plane across the middle of the area are shown. The smaller optimized arrays has higher resolution values than the conventional arrays, particularly at the middle and at depth. The larger optimized data set has an average resolution of 0.091 compared to 0.111 for the comprehensive data set with almost 100 times more arrays. The relative model resolution sections shows the differences more clearly.



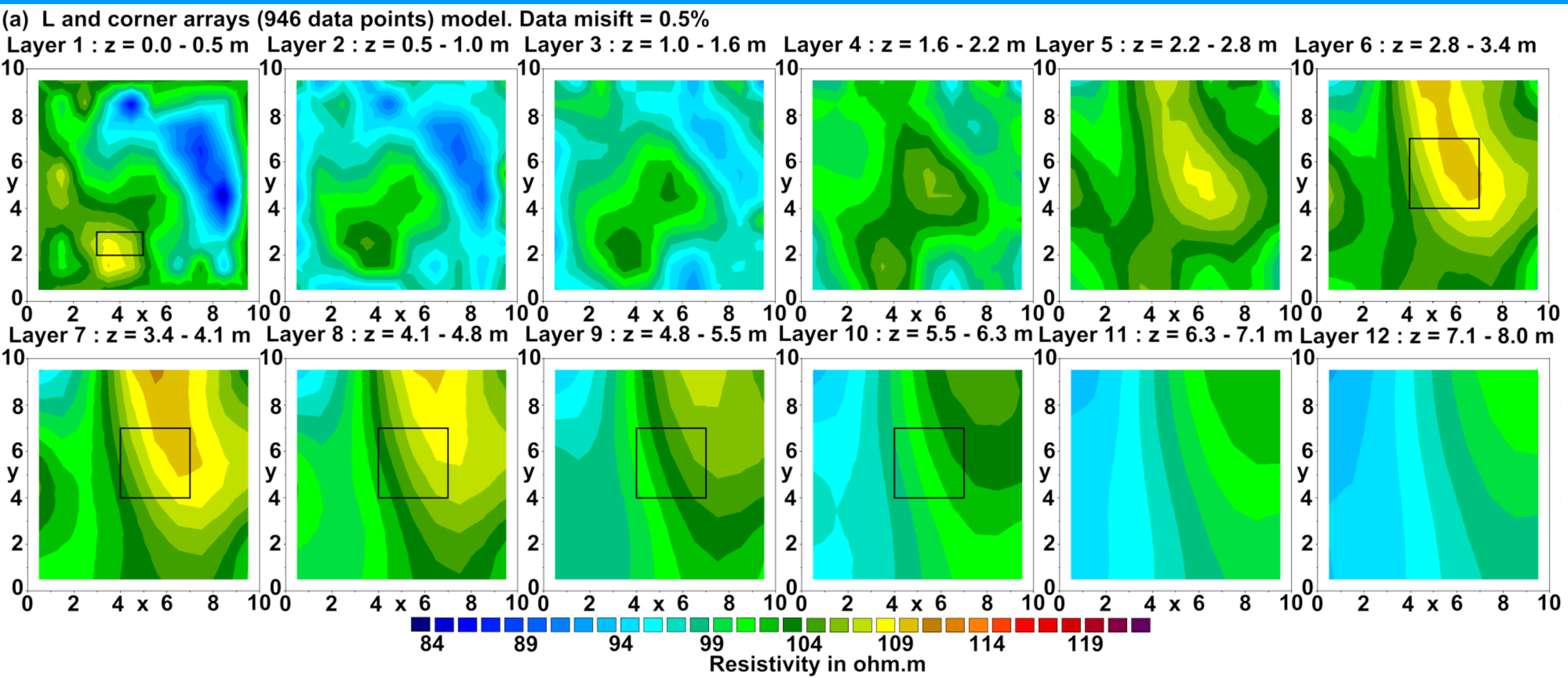
Test model

The test model consists of two blocks of 1000 ohm.m embedded within a 100 ohm.m medium. The data sets were generated for the conventional and optimized arrays (with electrode spacing of 1 m) with Gaussian random noise of 1 milliohm added.



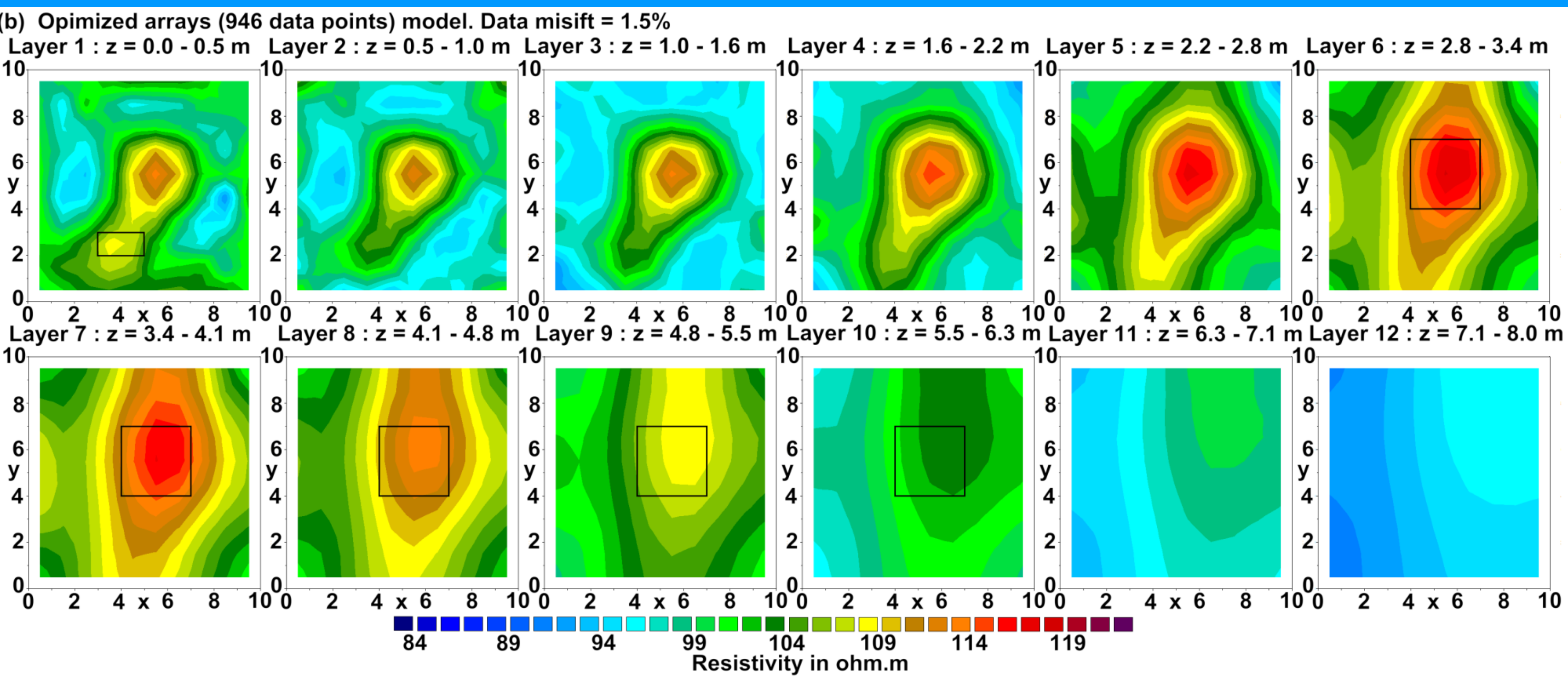
Inversion model – conventional arrays

The model is shown in the form of the horizontal sections. The 'L and Corner' arrays model detects the blocks with maximum resistivity values of about 110 ohm.m, slightly above the background of 100 ohm.m, but the shape of the deeper block is not clearly defined.



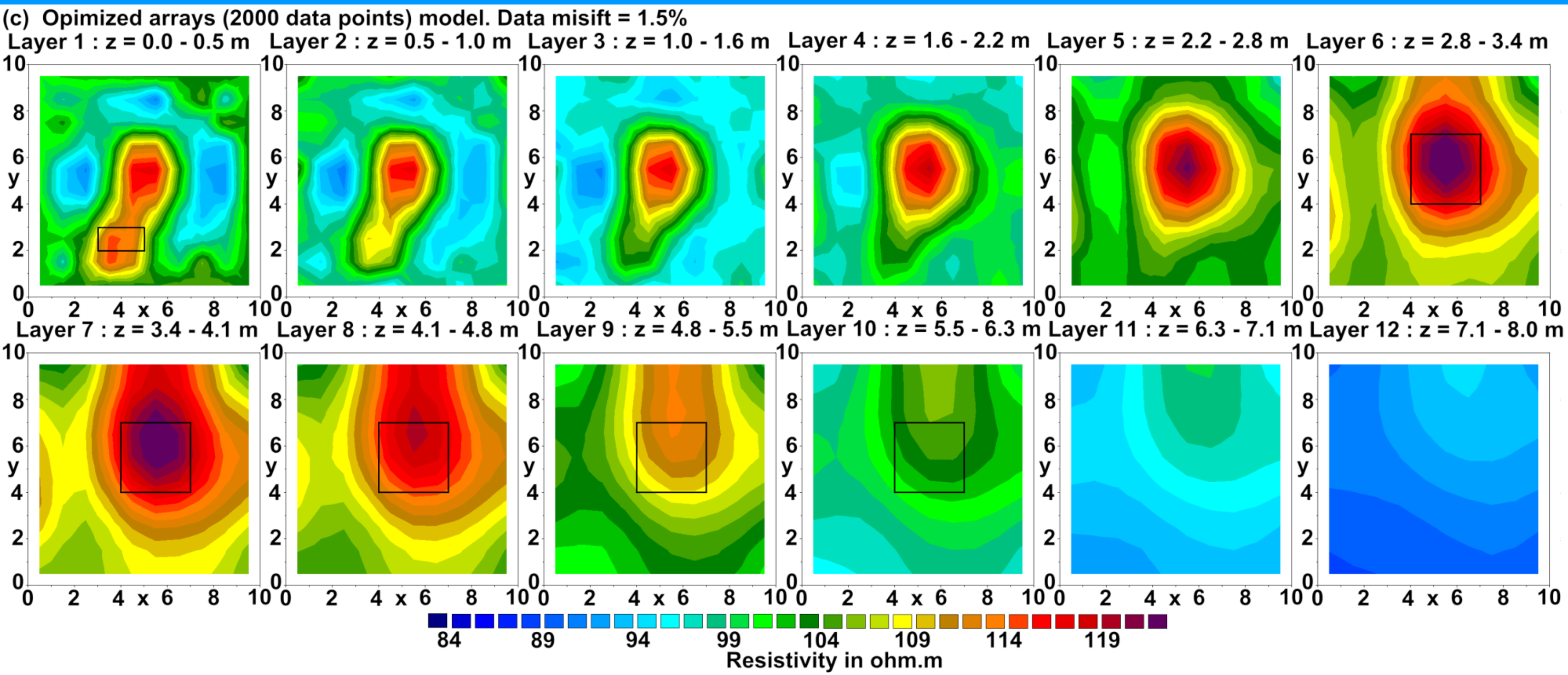
Inversion model – optimized arrays

The optimized data set model has a higher maximum value of 119 ohm.m for the deeper block with the maximum anomaly value closer to the true position in the upper part of the block. The top edge of the block is not well resolved with a small artifact that extends to the surface.



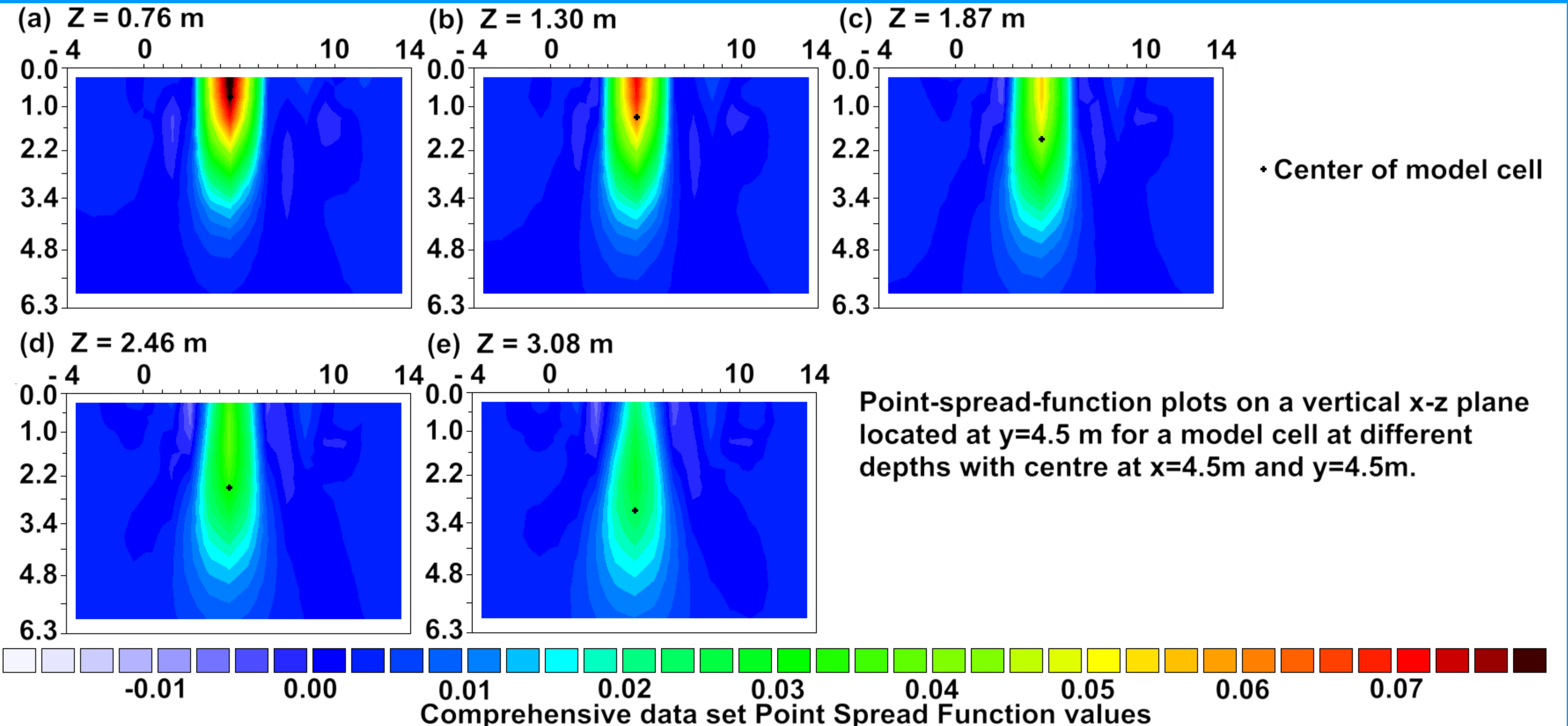
Inversion model – larger optimized data set

The model for the larger optimized set with 2000 data points shows higher values of 115 and 124 ohm.m at the positions of the top and bottom high resistivity blocks. However, the top edge of the deeper block is not well resolved with the anomaly extending to the surface.



Reason for poor vertical resolution

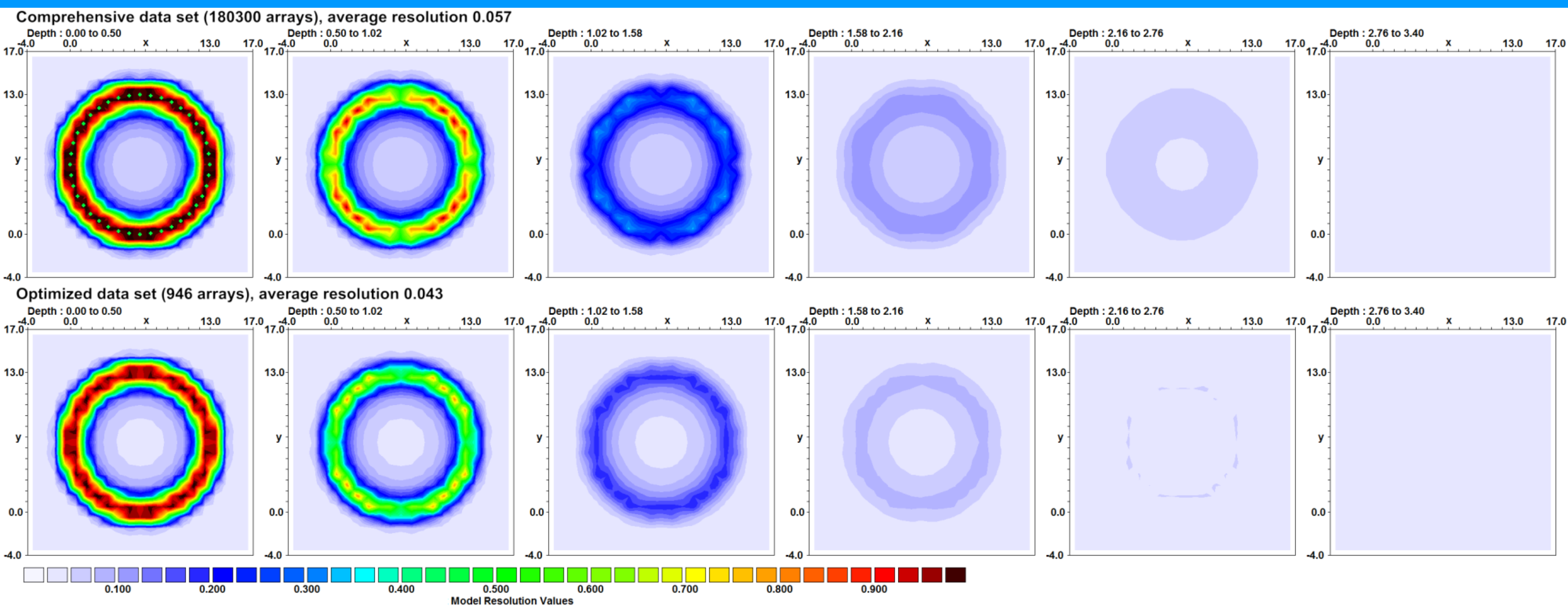
The poor vertical resolution is shown by the PSF plot which plots the off-diagonal elements of the resolution matrix. The PSF is elongated vertically showing that vertical resolution is poorer than horizontal resolution.



Point-spread-function plots on a vertical x-z plane located at y=4.5 m for a model cell at different depths with centre at x=4.5m and y=4.5m.

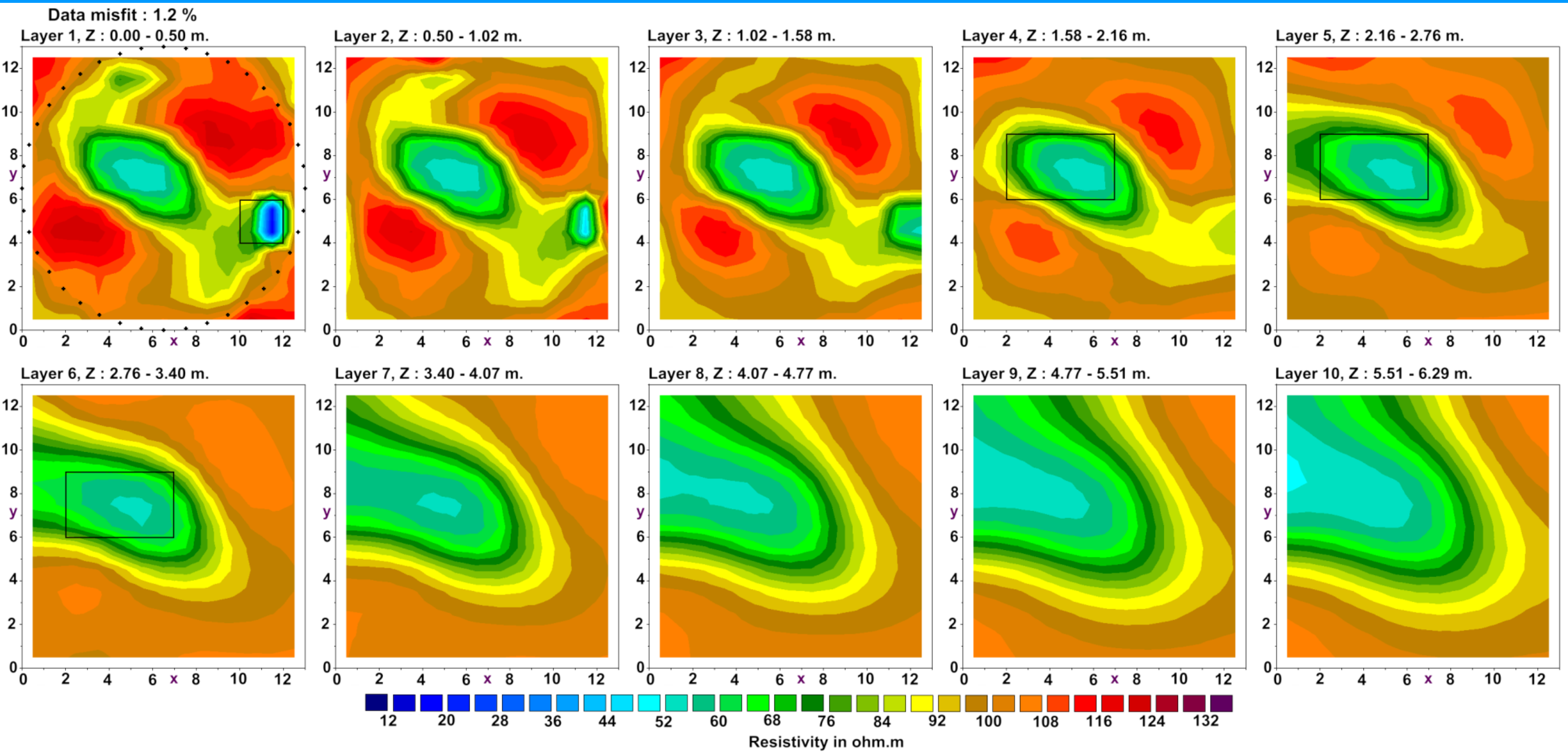
Non-rectangular perimeters

One advantage of the array optimization method that it can be used for perimeters of any shape. Below is an example of a circular perimeter with a diameter of 13 m. The depth of investigation is slightly less compared to a square grid with 10 m sides due to shorter maximum distance between two electrodes.



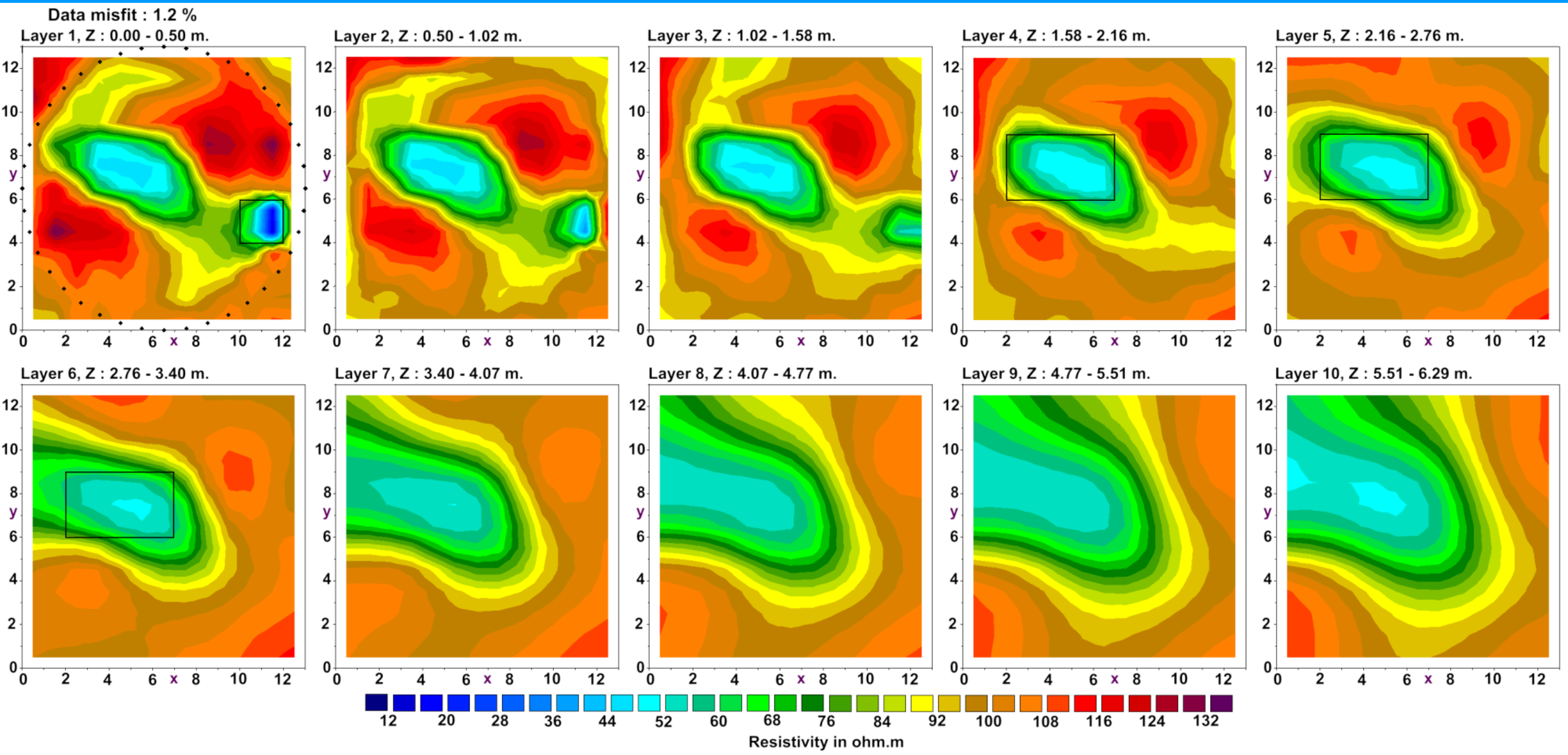
Circular perimeter test model – 946 data points

The model has 2 rectangular blocks of 10 ohm.m in a 100 ohm. medium. The 2 blocks are detected in the inversion model. However, the top and bottom boundaries of the deeper block are now well resolved.



Circular perimeter test model - 2000 data points

There is a slight improvement with the larger optimized data set. The left boundary of the top block and the bottom boundary of the lower block are better resolved.



Conclusions

(a) The arrays generated by the optimization routine has higher resolution compared to those created using heurestic algorithms. This is because they are selected to maximize the model resolution.

(b) The array optimization algorithm can be used for perimeters of any shape by setting limits of the maximum geometric factor and its senstitivity to position errors, and calculating the model resolution for the comprehensive data set of all viable arrays.

(c) For setups with less than 100 electrodes, the algorithm takes only minutes to create the optimized arrays on modern PCs.